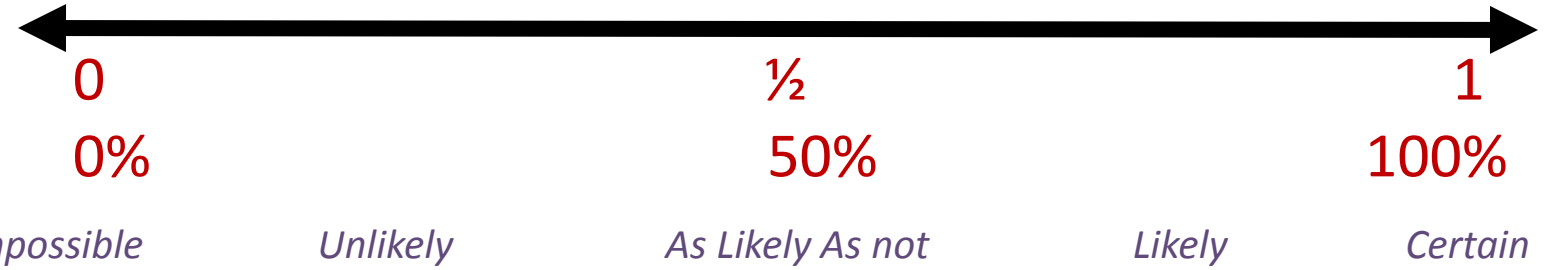
Probability Unit

Vocabulary and Concepts

Probability

- **Probability is a measure of how likely an event can occur.**
- **It is represented as a number between 0 and 1.**



- **This number can be a fraction, a decimal or a percent.**

$$P(O) = \frac{\text{Number of Orange pompoms}}{\text{Number of total pompoms}}$$

$$P(\text{outcome}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Sample Space, Tree Diagram and Fundamental Counting Principle

Sample space:

- The sample space of an experiment is the set of all possible outcomes of that experiment.
- The sample space of tossing a coin is: {head, tail}
- For any sample space, the SUM of all possible outcomes is 1.

Tree Diagram:

- A tree diagram is a visual representation of all possible combinations or outcomes of a given sample space.
- The tree diagram starts with one item (start point) that branches into two or more, each of which branch into two or more, and so on.
- It looks like a tree, with one trunk (start point) and multiple branches.

Fundamental Counting Principle:

A mathematical rule to figure out the total number of possible combinations or outcomes.

Example:

There are m ways to do one thing, n ways to do another. As per this rule, there are m * n ways of doing both.

2 pants * 3 shirts * 2 caps = 2*3*2 = 12 different ways you can build your outfit!

Draw a Tree Diagram for tossing a coin three times

First Toss = 2 choices

Second Toss = 2 choices

Third Toss = 2 choices

Sample Space:

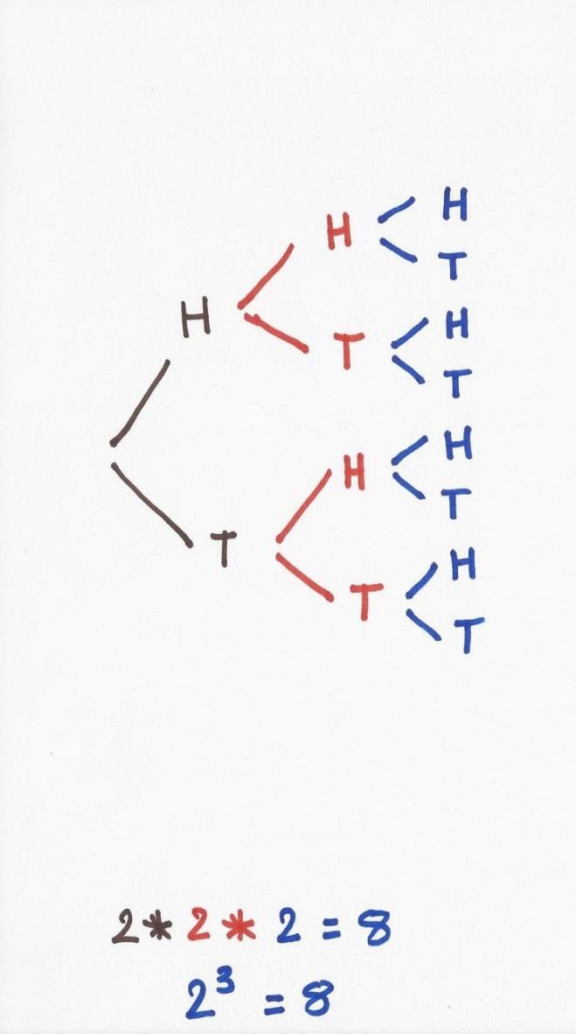
{ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

Fundamental Counting Principle :

- 2*2*2 = 8 total possible outcomes

-??Think?? If you toss a coin 14 Times, how many total possible Combinations or outcomes are there!!!

-Simple enough…2^{14} = _____



Charlotte is playing a board game. To move her game piece, she needs to roll the same number on two number cubes. Represent the sample space and find all the ways Charlotte could roll the same number using Tree Diagram

Sample space (list of all possible outcomes) of rolling a number cube twice:

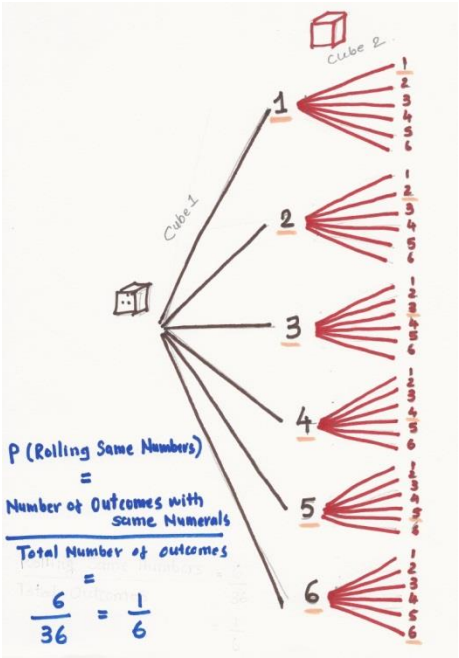
- **For our first roll (choice) – 6 possible outcome**
- **For our second roll (choice) – 6 possible outcome**

Total possible outcome using Fundamental Counting Principle

6 * 6 = 36

The list of all 36 outcomes (SAMPLE SPACE) can be obtained by making a list in a same manner you all are familiar with in Science Class !!!!! '**Punnett Square**'

	1	**2**	**3**	**4**	**5**	**6**
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6



Mutually Exclusive Events
- Events that can not happen at the same time

- Turning left and turning right are Mutually Exclusive (you can't do both at the same time)
- Cards: Kings and Aces are Mutually Exclusive

!!!Give it a try!!!

- Tossing a coin ?
- Turning left and scratching your head?
- Drawing King of Hearts?
- Choosing True/False?

Complement of an Event
- All outcomes that are NOT favorable events/results

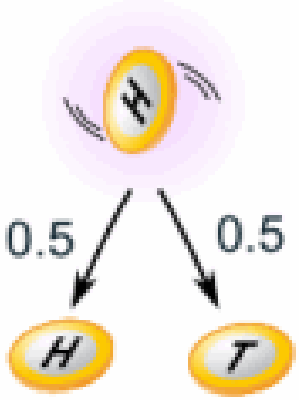
- When an event is {Heads}, the complement is {Tails}
- **When and event is {Monday, Wednesday} the complement is { ___, ___, ___, ____, ____ }**
- When an event is { Hearts}, the complement is { Spades, Clubs, Diamonds, Jokers}
- **Make one of your own complement event!**

Independent Events

- What happened in previous event will NOT affect the result of current event

You toss a coin and it comes up "Heads" three times ... what is the chance that **the next toss** will also be a "Head"?

- **The chance is simply ½ (or 0.5) just like ANY toss of the coin.**
- What it did in the past will not affect the current toss!



Binomial Probability

- Events or situations like ‘choosing true or false answers’ that have exactly two outcomes are Binomial Situations.
- The Probability of getting one of the two possible outcomes (true or false) is known as Binomial Probability.
- Think of more!!!.....

1) Head or Tail
2) Boy or Girl

Theoretical and Experimental Probability

Theory
A collection of ideas; a hypothesis to explain an outcome or event.

Experiment
A series of orderly trials carried out to verify the hypothesis by recording the results or observations.

Trial and Outcome
An experiment is a TRIAL and the result of an experiment is an OUTCOME.

Theoretical Probability
A probability that is predicted by analyzing a situation/scenario. Written as a ratio of…

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Experimental Probability
A probability that is determined by performing series of trials and recording observations. Written as a ratio of….

$$\frac{\text{Number of times an outcome occurred}}{\text{Total number of trials}}$$

Long Run Relative Frequency – Using the experimental data to make predictions

- The owner of the deli recorded the number of customers who ordered each of four sandwiches available. If the deli has 50 customers the first hour it is open, predict how many customers will order Turkey Sandwich! (Ready Book Problem)

1) Write Probability Definition….

$$P(\text{turkey Sandwich}) = \frac{\text{Number of Turkey Sandwiches}}{\text{Total Number of Sandwiches}} = \frac{180}{500}$$

2) Set Up a Proportion…..

$$\frac{180}{500} = \frac{x}{50}$$

Sandwich	Number of Customers
Ham	160
Cheese	100
Turkey	180
Veggie	60

3) Solve for X…

$500x = 180 * 50$

$500x = 9000$

$X = \frac{9000}{500}$

$X = 18$

4) Answer : 18 customers will order Turkey Sandwiches

Compound Probability of Independent Events

- **Two events, A and B, are independent if the fact that when A occurs, it does not affect the probability of B occurring.**
- **Compound Probability is P (A) AND P (B)**
- **P (A) AND P (B) = P (A) * P (B)**

Example…..

Landing on heads after tossing a coin **AND** rolling a 5 on a single 6-sided die.

P (Head) = $\frac{1}{2}$

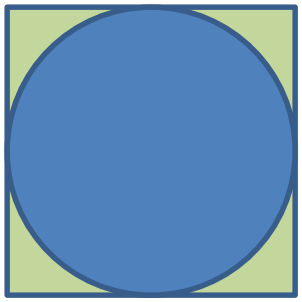
P (Rolling a 5) = $\frac{1}{6}$

P (Head **AND** Rolling a 5) also written as…. P (Head, Rolling a 5)

$= \frac{\mathbf{1}}{\mathbf{2}} * \frac{\mathbf{1}}{\mathbf{6}} = \frac{\mathbf{1}}{\mathbf{12}}$

Geometric Probability

- If Mary chooses a point in square, what is the probability that it is not in the circle? Radius = 6 units; ∏ = 3.14



  - If radius is 6, the side length of Square is 12.
  - Area of Square = $(Side)^2$ = 12 * 12 = 144 units squared
  - Area of circle = πr^2 = $(3.14)(6)^2$ = 113.04 units squared
  - The area of "not in a circle" = 144 – 113.04 = 30.96 units squared
  - P (not in a circle) = $\frac{area\ of\ not\ in\ a\ circle}{total\ area}$ = $\frac{30.96}{144}$ = 0.215
  - If asked in a % then multiply by 100 = 21.5 %

Addition Rule for "OR"

- When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event.
- P(A or B) = P(A) + P(B)

Example…

A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?

$P(2) = \frac{1}{6}$

$P(5) = \frac{1}{6}$

$P(2 \text{ or } 5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

Compound Probability of Dependent Events

- Two events are **dependent** if the occurrence of the first affects the occurrence of the second.

Example…

A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a king and the second card chosen is a jack?

P (King on first pick) = $\frac{4}{52}$

P (Jack on second pick given King on 1st pick) = $\frac{4}{51}$

P (King and Jack)= $\frac{4}{52} * \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$

Compound Probability of Dependent Events

There are 5 Red marbles, 4 Yellow marbles and 3 Green marbles in a jar. You picked a Red marble and gave it to me. What are your chances of picking a Red marble in second pick?

Let's Solve…

- 1st pick was one Red marble.
- Now we have 4 Red marble left. Total number of marbles in the jar now are…4 Red, 4 Yellow and 3 Green.

$$P(\text{Red}) = \frac{Red\ marbles}{Total\ marbles} = \frac{4}{11}$$

Probability and Factorial

- James, Melissa, Kevin and Lacy are ready to line up. How many different possible ways they can line up?

Understand….

There are 4 choices for 1st spot in line.

There are 3 choices for 2nd spot in line.

There are 2 choices for 3rd spot in line.

There is 1 choice for 4th spot in line.

- How many choices are there together?

(4 choices) * (3 choices) * (2 choices) * (1 Choice)

= 4 * 3 * 2 * 1

= 4!

= 24 different possible ways they can line up.